**The Fine Minimization Problem**

Suppose that a craftsman is given a set of tasks to complete. He cannot work on two or more tasks simultaneously, and he must complete each task once he has started it. Furthermore, he has agreed to pay a penalty for each task based on the time taken to begin that task. To be specific, let us say that the ith task will take a time ti to complete and require a penalty of pi for every unit of time that transpires before the initiation of the task. The problem is to determine the order in which the craftsman should work on the tasks in order to minimize his total fine, which is the sum of the task penalties.

We begin by considering the two-task case, say task 1, which will take time t1 to complete and require a penalty at a rate of p1, and task 2, which will take time t2 to complete and require a penalty at a rate of p2. The craftsman should first undertake whichever of the two tasks has the lower ratio of completion time to penalty rate, and in the case that the ratios are equal, he is free to start either task first. For example, suppose that t1/p1 ≤ t2/p2. This is equivalent to saying that t1p2 ≤ t2p1. But we recognize the two quantities on each side of this last inequality as the total fine for beginning task one first (the left side of the inequality) and the total fine for beginning task two first (the right side of the inequality). Thus the total fine is minimized when he begins the task having the lower ratio, and in the case of equal ratios, we see that the total fine will be the same regardless of the task order.

Now consider the general case of many tasks. Suppose for contradiction that some ordering of the tasks with minimum fine violates the initiation of the tasks according to an increasing ratio of completion time to penalty rate. Then there must be some first task in this ordering whose ratio exceeds the ratio of the next task. Imagine now that we switch the order in which these two tasks are done. The amount of the total fine resulting from tasks done before or after these two will be unchanged. In essence, we have now reduced the problem to the two task case, which we have earlier shown to have an optimal solution when the task with the lesser ratio is performed earlier. Therefore switching the two tasks will result in a lower fine.